AP Calculus Chain Rule Practice

① $f(x) = (2x+1)^3$

$f'(x) = 6(x+1)^2$

② $f(x) = (x^2+4)^{-2}$

$f'(x) = -2(x^2+4)^{-3}(2x)$

$= \dfrac{-4x}{(x^2+4)^3}$

③ $h(x) = \cos 2t^2 + 2\cos^2 t$

$h'(x) = (-\sin 2t^2)(4t) - 4\cos t \cdot \sin t$

④ $h(u) = (3u^2+5)^3(3u-1)^2$

$h'(u) = (3u^2+5)^3 2(3u-1)\cdot 3 + 3(3u^2+5)^2 \cdot 6u(3u-1)^2$

$= 6(3u^2+5)^2(3u-1)[3u^2+5 + 3u(3u-1)]$

$= 6(3u^2+5)^2(3u-1)(12u^2-3u+5)$

⑤ $f(y) = \left(\dfrac{y-7}{y+2}\right)^2$

$f'(y) = 2\left(\dfrac{y-7}{y+2}\right)\left(\dfrac{(y+2)-(y-7)}{(y+2)^2}\right)$

$= \dfrac{18(y-7)}{(y+2)^3}$

⑥ $f(r) = (r^2+1)^3(2r+5)^2$

$f'(r) = (r^2+1)^3 2(2r+5)2 + 3(r^2+1)^2 \cdot 2r(2r+5)^2$

$= 2(r^2+1)^2(2r+5)[2(r^2+1) + 3r(2r+5)]$

$= 2(r^2+1)^2(2r+5)(8r^2+15r+2)$

⑦ $f(x) = 4\cos(\sin 3x)$

$f'(x) = -4\sin(\sin 3x)\cdot\cos(3x)\cdot 3$

$= -12\cos(3x)\cdot\sin(\sin 3x)$

⑧ $g(x) = (1+4x^2)^{1/2}$

$g'(x) = \frac{1}{2}(1+4x^2)^{-1/2}(8x)$

$= \dfrac{4x}{\sqrt{1+4x^2}}$

⑨ $g(y) = (25-y^2)^{-1/2}$

$g'(y) = -\frac{1}{2}(25-y^2)^{-3/2}(-2y)$

$= \dfrac{y}{(25-y^2)^{3/2}}$

(10) $f(x) = (\sin 3x)^{-1/2}$

$f'(x) = -\frac{1}{2}(\sin 3x)^{-3/2} \cdot \cos(3x) \cdot 3$

$= \dfrac{-3\cos 3x}{2(\sin 3x)^{3/2}}$

(11) $f(x) = (2x^3 - 5x^2 + x)^{1/3}$

$f'(x) = \frac{1}{3}(2x^3 - 5x^2 + x)^{-2/3}(6x^2 - 10x + 1)$

$= \dfrac{6x^2 - 10x + 1}{3(2x^3 - 5x^2 + x)^{2/3}}$

(12) $h(y) = \cos\sqrt{y^2+1}$

$h'(y) = -\sin\sqrt{y^2+1} \cdot \frac{1}{2}(y^2+1)^{-1/2} \cdot 2y$

$= \dfrac{-y \cdot \sin\sqrt{y^2+1}}{\sqrt{y^2+1}}$

(13) $g(x) = (1 + \cos^2 2x)^{1/2}$

$g'(x) = \frac{1}{2}(1 + \cos^2 2x)^{-1/2}(2\cos 2x)(-\sin 2x)(2)$

$= \dfrac{-2\sin 2x \cdot \cos 2x}{\sqrt{1 + \cos^2 2x}}$

(14) $f(x) = \cos^2 x^2$

$f'(x) = 2\cos x^2 \cdot -\sin x^2 \cdot 2x$

$= -4x \cdot \cos x^2 \cdot \sin x^2$